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RATIO OF CROSS SECTIONS OF ELEMENTARY STRONG REACTIONS USING CLEBSCH GORDON (CG) CO-EFFICIENTS

Tadasha Dash* & Anjana Acharya
*Department of Physics, Veer Surendra Sai University of Technology, Burla Odisha India-768018

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ABSTRACT

An The strong reactions having certain properties obeying the conservation laws of isospin, baryon number, lepton number, strangeness, hypercharge etc are considered. The clebsch Gordon coefficients for SU(3) symmetry group satisfying orthogonal properties are expressed. The cross sections are found out in terms of these co-efficients.

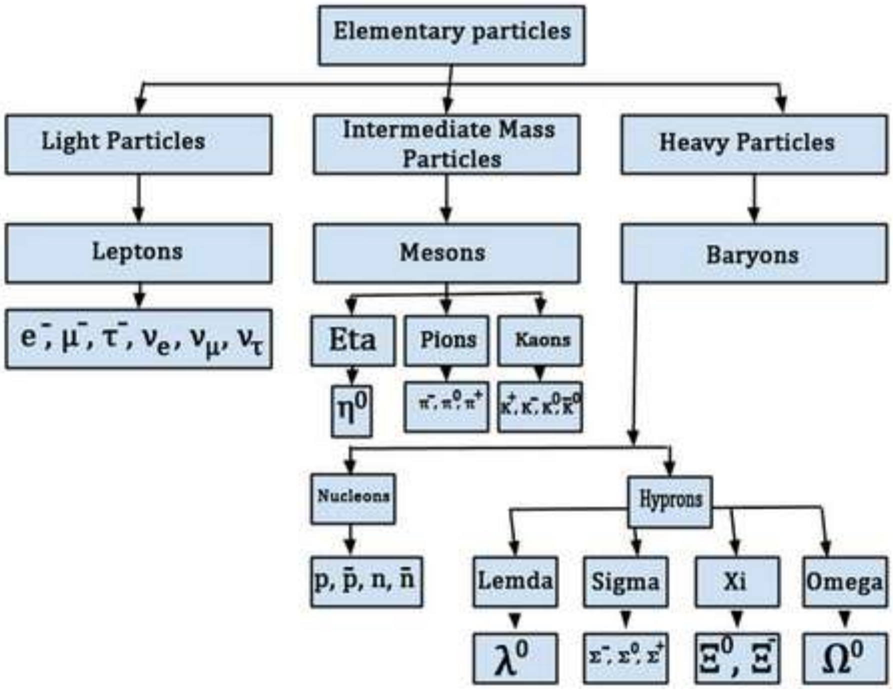
KEYWORDS: strong reactions, conservation laws, CG co efficient, orthogonality condition

I. INTRODUCTION

The branching ratios of the strong reaction plays an important role in studying astrophysics as well as nucleosynthesis.Th CG co-efficients are used to get the cross sections of the elementary particles from intermediate to high energy nuclear reactions.The total cross section for p +p → p p π^0 was measured at centre of mass energy from 1.5 to 23 Mev above threshold at Indiana cooler.The measured energy dependence of the total cross section is not compatible with the low partial wave values[1,2].

II. REVIEW WORKS

Classification of the elementary particle particles is as shown below



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Conservation laws [3-4]

The behaviour of the elementary particles is restricted by number of conservation laws. The most familiar physical in large scale experiments that are conserved in all (strong, electromagnetic and weak) interactions are:

Conservation of linear momentum

The total linear momentum is conserved in all types of interactions. This is related to invariance of physical laws under translation in space (homogeneity of space) implying that laws of interaction are independent of the place of measurement.

Conservation of angular momentum

The conservation of angular momentum includes both types (orbital and spin) of angular momentum together but the orbital and spin angular momenta may not be separately conserved.The first is given by the motion of the object as a whole about any chosen axis of rotation. The second is the intrinsic angular momentum of each object about an axis through its own centere of mass. This is also valid for all type of interactions and is releated to the invariance of physical laws under rotation (isotropy of space).This follows from the Hamilton's equations in classical mechanics, if we write in terms of angular coordinate (θj) and conjugate momenta (L_i). In quantum mechanics angular momentum J must commute with H. [J H] =0. Here if angular momentum is constant of motion than H remains invariant under the operations.

Conservation of energy

This holds for all type of interaction It is related to the invariance of the physical laws under translation along the time axis. This means that laws of interaction do not depend on time measurement. Conservation of energy is complicated with elementary particles because a large fraction of total energy is interchanged between rest energy associated with mass and kinetic or potential energy. The sum of these three, the total energy is conserved in any reaction.

Conservation of charge (C)

The charge is conserved in all processes and no exceptions are known.It is due to conservation of charge that the electron cannot decay. This is related to the gauge invariance of the electromagnetic field. Like $n \rightarrow p^+ + e^- + \nu_e$

In this equation the total initial charge is equal to total final charge. If we consider the decay of electron into photons and neutrinos then it would violate the law of conservation of charge. Electron be the lightest charge particle, the conservation of charge implies that the electron must be a stable particle.

Conservation Of Baryon Number(B)

The number of baryons minus number of anti baryons is conserved.The law of conservation of B was introduced to forbid the decay of proton to positron and photon. The baryonic charge or the baryon number is an additive quantum number. All baryons (p, n, Σ^+ ,Σ^- ,Σ^0 ,Ξ^- ,Ξ^0 ,Λ_0) are assigned a baryon quantum number B = +1. Anti baryons of (Σ^+ Σ^- Ξ^- Ξ^0 Λ^0) are assigned a baryon number B = -1, while the non-baryons photon leptons and mesons are assigned a baryon number B = 0. Since the baryon number is an additive quantum number the baryon number of a set of particles is the sum of the baryon number of each particle in the set. According to this law of conservation in any reaction the initial baryon number is the same as the final baryon number. So the total number of baryons and anti-baryons should remain constant for all the interaction. Thus from examples

$$p \rightarrow \pi^+ + \pi^0$$
$$B : +1 \quad 0 \quad 0$$

is forbidden by baryon conservation, while

$$\Delta^{++} \rightarrow p + \pi^+$$
$$B : +1 \quad +1 \quad 0$$

is allowed by baryon number conservation as well as by all other conservation laws. This reaction involves only hadrons and it occurs with the characteristic time of 10^{-23} second. Similarly $n \rightarrow p + e^- + \nu_e$

$$B : 1 \quad 1 \quad 0 \quad 0$$

is also allowed by baryon number conservation. Since it involves leptons as well as hadrons, the reaction goes at a much slower rate (the neutron mean life being 1000 seconds).

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Conservation of lepton number (L)

The leptons (e^- μ^- ν_e ν_μ) and the anti-leptons are (e^+ μ^+ ν_e ν_μ) are respectively assigned L = +1 and L = -1. The non leptons (baryons and mesons) are assigned L = 0.According to the law of conservation of lepton number the initial lepton number must be equal to the final lepton number.Net lepton number in any process always remains conserved e.g.

$$e^- + e^- \rightarrow \pi^- + \pi^-$$

At high energies, this reaction is kinematically allowed, and it certainly satisfies charge conservation, but it is not observed. Naturally, lepton number conservation would prevent this process from taking place. In fact, reactions such as

$$\mu^- \rightarrow e^- + \gamma$$
$$\mu^- \rightarrow e^- + e^+ + e^-$$

although kinematically allowed, have also never been observed. Thus, the electron and its neutrino have an electron-lepton number $Le = 1$, whereas the other leptons have $Le = 0$. The muon and its neutrino have muon-lepton number $L_\mu = 1$, whereas the other leptons have $L_\mu = 0$, and similarly for the τ-lepton and its neutrino. The net lepton number of any particle can therefore be expressed as the sum of the electron number, the muon number, and the τ-lepton number.

Conservation of isospin

The isotopic spin or isospin is a quantum number or variable applied to the strongly interacting fundamental particles, the hadrons and their compounds, which facilitate consideration of consequences of the charge independence of the strong nuclear forces. Though isospin has mathematical properties similar to that of the spin but there is no direct physical relationship between them.it is used to identify related energy levels or quantum states of isobars (systems with same number of nucleons). Heisenberg was first to apply a quantum number to charge and referred to the concept as "isospin**".**

A multiplet number M is therefore assigned to such particles to indicate the number of their different charge states. For instance, for nucleons (protons and neutrons) M = 2, for pions M = 3, for kaons M = 2 etc. As in multiplicity of atomic energy states due to spin the total number of states is (2n+1) where s is spin quantum number of electron. Now isospin quantum number I from the relation M =2I+1. Isospin is treated as vector of magnitude (I + 1) but it is dimensionless. Its component in Z axis is given as Iz. Which has the allowed values I, (I – 1),(I – 2), (I - 2)…. –I. For nucleons (M = 2) I =(M – 1)/2 =1/2, with the two components $I_3 = +1/2$ assigned to proton and $I_3 = -1/2$ assigned to neutron. Isospin is conserved in strong interaction, but is violated in electromagnetic and weak interaction. The Z- components of isospin I_3 is conserved in strong and electromagnetic interactions, but not in weak interaction.

The charge Q of a state and the value of I_3 for this state are connected by the Gell-Mann-Okubo relation, $Q = I_3 + \frac{1}{2} B$ where B is the baryon number.

The addition of two isospins follows the addition of two angular momenta exactly.

The reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$, does conserve I-spin. The right side has I-spin equal to 1/2 (I = 0 and I = 1/2 have only one possible sum). The left side is the resultant of I = 1 and I = 1/2. Therefore, the reaction can go via the strong interaction because isotopic spin is conserved both the left side and the right side can have I = 1/2.

***Conservation of hypercharge** (Y)*

Hypercharge is the sum of the baryon number B and strangeness S. since baryon number is always conserved, Y is a good quantum number conserved in strong interactions. Thus charge is related with isospin,baryon number and strangeness as

$$\mathbf{Q = I_3 + \frac{1}{2}} Y = I_3 + \frac{1}{2} B + \frac{1}{2} S$$

This is called Gell-Mann-Nishijima relation. In strong and electromagnetic interactions, the hypercharge is conserved but not in weak interactions. Let us take a reaction

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$$\mathbf{P + p} \rightarrow \Lambda^0 + K^0 + p + \pi^+$$

$$Y: 1 + 1 \rightarrow \; 0 + 1 + 1 + 0 \quad \Rightarrow \Delta Y = 0$$

Since hypercharge is conserved it is a strong interaction collision.

***Conservation of strangeness** (S)*

The strangeness quantum number is first proposed by Pais and developed by Gell-Mann and Nishijima, to understand the strange behaviour of K-mesons and hyperons (strange particles). S is defined as the difference of the hypercharge Y and the baryon number B.Strangeness is conserved in strong and electromagnetic interactions but not in weak decay. For an event involving a weak interaction, S however can change but by not more than ±1. Consider the hadron Λ^0 which is a baryon (the Λ^0 has a rest energy of 1116 MeV compared to 938 MeV for the proton). The Λ^0 is produced via Strong Interactions such as

$$\pi^- + p \rightarrow \Lambda^0 + K^0$$

and

$$\pi^0 + p \rightarrow \Lambda^0 + K^+$$

where the K^0 and K^+ are mesons. The K's are B = 0 particles as are the pions, so baryon number is conserved in the reaction since the Λ^0 has B = 1. So Λ^0 must be a hadron (it's produced in a reaction which goes at the characteristic rate of strong interactions) and it is a baryon. The Λ^0 is unstable and decays via $\Lambda^0 \rightarrow p + \pi^-$. This reaction involves only hadrons and baryon number is conserved, so it should go at the SI (strong interaction) rate of $\propto 10^{-23}$ seconds. In fact the observed Λ^0 lifetime is much longer, about 10^{-10} seconds, which indicates that the reaction occurs via the WI (weak interaction) and not the SI. This is not an isolated case.

Conservation of parity

This holds for strong nuclear and electromagnetic interactions but is violated in weak interactions. It is related to the invariance of physical laws under inversion of space coordinates so that x, y, z can be replaced by –x, -y, -z. This is equivalent to combined reflection and rotation. For Parity to be conserved, the parity operator must commute with the Hamiltonian. Spatial parity depends on the orbital angular momentum and is given by $P = (-1)^1$ where l is azimuthal quantum number. Thus the states of even l has even parity (P = +1) and the states of odd l has odd parity. (P = -1). The two successive parity operations bring the system back to its initial state so that $P^2 = 1$ which gives us the two possible eigen values ±1 for P. The parity of a particle is usually designated by the symbols (+) for even and (-) for odd written as superscripts above the total angular momentum, thus (J_p). Example for nucleons we write (J_P) = ($½^+$) for pions (J_P) = (0^-)

Charge conjugation

Charge conjugation is the operation which changes the sign of charge of particle without affecting any of the properties unrelated to charge. It may be defined as the transformation between a particle and its anti particle. Charge conjugation means reversal of signs of all types of charges i.e. electronic, baryonic, and leptonic of the particle. If a physics law holding for a particle then it holds for corresponding antiparticle also then the principle of charge conjugation is said to be valid.. The principle of charge conjugation implies that the cross-section of the reactions of a given energy must be the same. Strong and electromagnetic interactions are invariant under charge conjugation but weak interaction does not obey charge conjugation.

Time Reversal (T)

The operation T means time-reversal means which reverses the direction of time, or the direction of all motions.under this operation displacement, acceleration , and electric field remain invariant but momenta and magnetic fields invert their signs. Time reversal invariance find its simplest application in the world of particles, where it appears to govern the strong and electromagnetic interactions and possibly the weak. The time reversal process is the creation of an electron-positron pair by the collision of two photons. All the known fundamental equation of motion are invariant in time reversal. Strong and electromagnetic interactions are invariant under time reversal transformation, but weak interaction is not.

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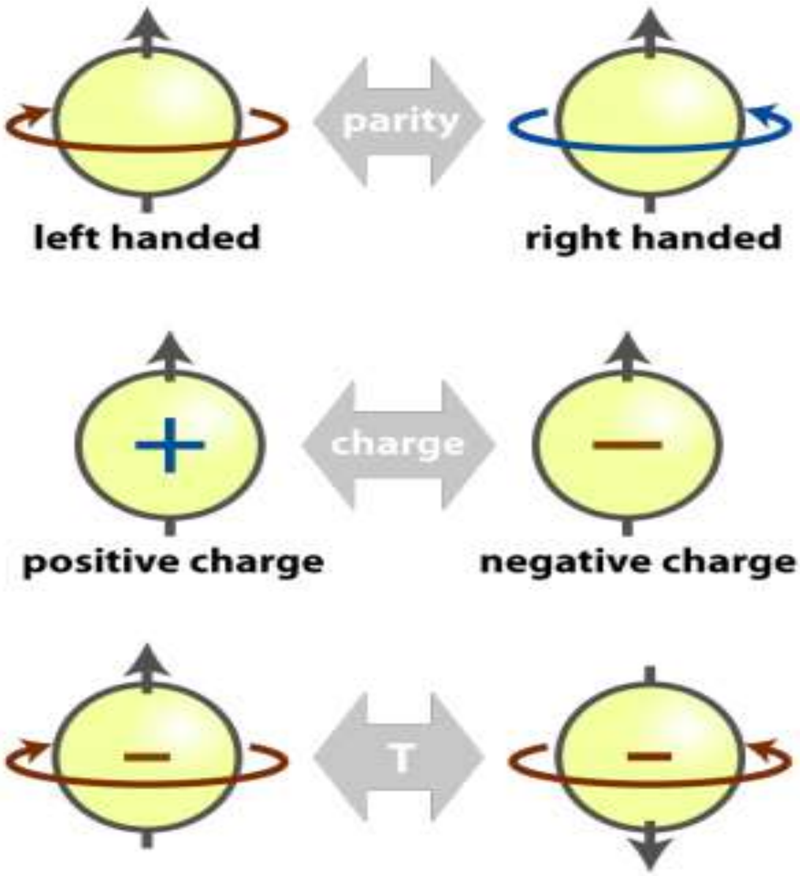
Symmetry		Conservation law
Translation in time	↔	Energy
Translation in space	↔	Momentum
Rotation	↔	Angular momentum
Gauge transformation	↔	Charge

Some Symmetries And The Associated Conservation Laws

III. CPT THEOREM

CPT Theorem states: "All interactions described by a local Lorentz invariant gauge theory must be invariant under the combined operation CPT "

CPT violation would imply non-locality and/or loss of Lorentz invariance. Impossible to write down relativistic quantum field theories. Impossible to describe interactions in terms of Feynman diagrams. CPT conservation implies that CP violation is equivalent to T violation The Universe needs CP violation for the matter-antimatter asymmetry and it needs T violation for the arrow of time.



The action of the P (top), C (middle) and T (bottom) operators on a fundamental fermion. the action of the operator twice on the state gives the original state back

The weak interactions are not invariant under the parity transformation ***P;*** the cleanest evidence for this is the fact that the antimuon emitted in pion decay $\Pi^+ \rightarrow \mu^+ + \nu_\mu$

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always comes out left-handed. Nor are the weak interactions invariant under C, for the charge-conjugated version of reaction would be $\pi^- \rightarrow \mu^- + \bar{\upsilon}_\mu$

with a left-handed muon, whereas in fact the muon always comes out right handed.

However, if we combine the two operations we're back in business: CP turns the left-handed antimuon into a right-handed muon, which is exactly what we observe in nature. Many people who had been shocked by the fall of parity were consoled by this realization; perhaps it was the combined operation that our intuition had been talking about all along-maybe what we should have meant by the "mirror image" of a right-handed electron was a left-handed positron.

There are four different types of fundamental interactions in nature, which govern the behaviors of all observable physical system. These are gravitational, electromagnetic, weak and strong nuclear interactions. The gravitational forces are not significant for elementary physics. Out of rest three, weak force has a very short range (< 10^{-17}m) and is extremely feeble compared to strong and electromagnetic forces.

Gravitational force acts between all bodies having masses. It is described by the long range inverse square type Newtonian laws of gravitation, later broadened by Einstein in his General Theory of Relativity, which describes the gravitational interaction in terms of the curvature of space. So far as the elementary particles are concerned, the gravitational interaction between them can be entirely left out of consideration, because it is very weak compared to other interactions.

Electromagnetic interaction is much stronger than the gravitational interaction. It is also a long range inverse square type interaction. Its strength is determined by Sommerfeld‟s fine structure constant $\alpha = e2/4\ \pi\varepsilon_0$~1/137. The electromagnetic interaction between a proton and an electron is about 1037 times stronger than the gravitational force between them at the same distance. In modern quantum field theory, the electromagnetic interaction between the charged particles is described in terms of the exchange of virtual protons, which are quanta of this field. The electromagnetic interaction is manifested in the chemical behaviours of the atoms and molecules, Rutherford scattering and so forth.

The third fundamental interaction is the weak nuclear interaction, which is responsible for the nuclear beta-decay and the weak decay of certain elementary particles like the muons, pions, k-mesons and some hyperons. Unlike gravitational or electromagnetic interaction the weak interaction is a very short range force.The coupling constant of the weak interaction has a value g=1.4*10^{-62} J-m3. When expressed indimensionless form the weak interaction constant gw is found to be small compared to the fine structure constant α~1/137. The weak interaction is mediated through the heavy vector bosons, known as $W\pm\ and\ Z_0$ bosons discovered by G Amison et al., using p p colliding beam. An important characteristic of the weak interaction which distinguishes it from the other fundamental interactions is that parity is not conserved in weak interaction.

The fourth fundamental interaction is the **strong nuclear interaction** between protons and neutrons. The range being h/$m\pi$ c~ $10^{-15}m$. Within this range it predominates over all the other forces between the neutron and the proton with the characteristic strength parameter a~1/137 of the electromagnetic interaction. The time for the operation of the strong interaction is of the order of the characteristic nuclear time (~10^{-23}s). The strong inter nu

Interaction	Characteristic Constant	Strength	Range of interaction	Typical cross section	Typical lifetime
Strong	g^2h/hc	1~10	10^{-13} cm	10^{-26} cm^2	10^{-23} s
Electromagnetic	e^2/hc	1/137	∞	10^{-29} cm^2	10^{-16} s
Weak	$g_w^2/hc = G_F m_p^2 c/h^3$	10^{-5}	10^{-16} cm	10^{-38} cm^2	10^{-10} s
Gravitational	Gm_p^2 /hc	10^{-39}	∞		

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Particle	mass (MeV)	charge	spin	stability	interaction
(a) Leptons					
e^-	0.511	-1	1/2	stable	weak, electromagnetic
μ^-	106	-1	1/2	unstable	weak, electromagnetic
τ^-	1777.1	-1	1/2	unstable	weak, electromagnetic
ν_e	??	0	1/2	stable	weak
ν_μ	??	0	1/2	stable	weak
ν_τ	??	0	1/2	stable	weak
(b) Quarks:					
u	3	2/3	1/2	-	weak, electromagnetic, strong
d	6	-1/3	1/2	-	weak, electromagnetic, strong
c	1300	2/3	1/2	-	weak, electromagnetic, strong
s	100	-1/3	1/2	-	weak, electromagnetic, strong
t	175000	2/3	1/2	-	weak, electromagnetic, strong
b	4300	-1/3	1/2	-	weak, electromagnetic, strong
(c) Gauge Bosons:					
g	0	0	1	stable	strong
γ,	0	0	1	stable	electromagnetic
$W^\pm$	80400	$\pm$	1	unstable	weak
Z	91187	0	1	unstable	weak

IV. PRODUCTION OF PIONS[5]

Pions are particles with mass intermediate with that of an electron and proton. Their existence had been predicted by Yukawa to explain the strong short range inter nucleon force. Yukawa predicted that pions are unstable. Pions are subsecuntly discovered in the cosmic rays by C.F.Powell and others. Pions are produced in the N-N collisions under the influence of strong nuclear interaction. After their discovery in the cosmic rays C.G.M. Lattes and E. Gardener were able to produce pions in the laboratory by using the 380 MeV preoton beam obtained from the 4.67m synchro-cyclotron at the Berkeley Lawrence Lanoratory.Thus π^+ and π^- mesons are produced in the following reactions.

$$\mathbf{p+p} \rightarrow p + n + \pi +$$
$$\mathbf{p+n} \rightarrow p + p + \pi -$$

mass of pions: The rest mass of the negative pion is $M\pi^- = (273.2 \pm 0.1)\, m_e$
$= 139.577 \pm 0.013$ MeV/c^2

The positive pions have the same mass. The mass of neutral pion is found to be
$M\pi^0 = 134.971 \pm 0.019$ MeV/c^2

Decay of pions: Unless pions have sufficiently high energies (~150MeV), positive pions do not interact to any great extent with nuclei and decay at rest according to the reaction, $\pi + \rightarrow \mu^+ + \nu_\mu$

Pions may also decay according to the reactions

$$\pi^+ \rightarrow e^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$
$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$
$$\pi^0 \rightarrow e^+ + e^- + \gamma$$
$$\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$$

It is evident from the nature of the products that π^0- mesons decay by the electromagnetic interaction while charged pions decay by strong and the weak interactions both.

K- Mesons Production

K-mesons are produced along with the pions when high energy protons bombarded a suitable target. With increasing proton energy the fraction of K-mesons relative to pions increases. There are essentially four K-mesons i.e, two uncharged mesons k^0 and $\bar{k}^0$ which are particle and anti-particle of one another, and two charged mesons k^+ and k^-, which are particle and anti-particle of one another.These are obtained in laboratory by the high energy protons or pions with various atomic particle.

Mass of kaons: The adopted values of the mass of kaons are:

$$M_{k^+} \text{ or } M_{k^-} = 493.67 \pm 0.0015 \text{ MeV/C}^2$$
$$M_{k^0} \text{ or } M_{\bar{k}^0} = 497.67 \pm 0.13 \text{ MeV/C}^2$$

k- meson (k^+ and k^0) are strange particles having strangeness quantum number +1.

V. HYPERONS

Hyperons are semistable particles having masses greater than that of the nucleon. The name hyperon was taken from Greek word hyper, "meaning above". Hyperon is the collective name of baryons with nonzero strangeness number S. The hyperons have long lifetime relative to 10^{-22} sec and decay by photon emission or through weaker decay interactions. The known hyperons are $\Sigma+$, $\Sigma-$, Σ_0, $\Xi-$, Ξ_0 , Λ_0 and Ω^- and their anti-particles.

Λ-Hyperon

These particles are observed in the tracks of two charged particles emerging from a point in the gas of a cloud chamber. This unstable neutral particle was named as lambda because of the characteristic appearance of its decay in a cloud chamber.

The present best estimated mass of Λ^0 is $M_{\Lambda}{}^0 = 1115.60 \pm 0.05$ MeV/C^2

Λ-hyperon undergoes following decay reactions:

$$\Lambda^0 \rightarrow p + \pi^-$$
$$\Lambda^0 \rightarrow n^0 + \pi^0$$

Σ-HYPERSONS

It is the family of hyperons with the greatest number of members.

Mass of Σ^+ = 1189.36 ± 0.06 MeV/ C^2

Mass of Σ^- = 1197.34 ± 0.05 MeV/C^2

Mass of Σ^0 = 1192.46 ± 0.08 MeV/C^2

Its decay scheme is $\Sigma^+ \rightarrow p + \pi^0 + 116.1$ MeV (~51%)

$\Sigma^+ \rightarrow n + \pi^+ + 110.3$ MeV (~49%)

$\Sigma^- \rightarrow n + \pi^- + 118.5$ MeV

$\Sigma^0 \rightarrow \Lambda^0 + \gamma + 80$ MeV

Ksi-hyperons

The Ξ's are originally called cascade particles, because a Ξ-particle decays to produce a π-meson and a Λ^0 meson, which in turn decays into a nucleon and another π-meson.

Modes of decay: $\Xi^- \rightarrow \Lambda^0 + \pi^-$ +66.3MeV

$\Xi^0 \rightarrow \Lambda^0 + \pi^+$ + 70MeV

Mass of Ξ^0 = 1314.3 ± 1.0MeV

Mass of Ξ^- = 1320.8 ± 0.2 MeV

VI. CG CO- EFFICIENT

In physics, the **Clebsch–Gordan** (**CG**) **coefficients** are numbers that arise in angular momentum coupling in quantum mechanics. They appear as the expansion coefficients of total angular momentum eigen states in an uncoupled tensor product basis. In more mathematical terms, the CG coefficients are used in representation theory, particularly of compact Lie groups, to perform the explicit direct sum decomposition of the tensor product of two irreducible representations (i.e., a reducible representation) into irreducible representations, in cases where the numbers and types of irreducible components are already known abstractly. The name derives from the German mathematicians Alfred Clebsch and Paul Gordan, who encountered an equivalent problem in invariant theory.The coupled states can be expanded via the completeness relation (resolution of identity) in the uncoupled basis.

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The present works

Consider two reactions

A. $p + p \rightarrow \pi^{+} + d$
B. $n+p \rightarrow \pi^{0} + d$

Lets see this reaction is allowed or forbidden. In order to verify this the conservation laws are conserved or not in both the reactions.

- Charge is conserved.
- Baryon number is conserved.
- Lepton number is conserved.
- Strangeness is conserved.
- Isospin is conserved.

So both the reactions are allowed and are strong reactions.

In order to find their cross section in terms of isospin in matrix form and branching ratio we have to use cg coefficients.The difference in cross section between $pp\rightarrow \pi^{+}d$ and $np\rightarrow \pi^{0}d$ relates to isospin only. Using coupling presentation for isospins and noting the orthogonality of the isospin wave functions.

Isospin can be represented by a set of operators which not only obey the same algebra as the components of angular momentum, but also couple in the same way as ordinary angular momenta.

Since nuclear force does not depend on the electric charge, we can consider the proton and neutron to be separate manifestations of the same particle, the nucleon. The nucleon may thus found to be in two different states: a proton and a neutron. In this way, as the protons and neutrons are identical particles with respect to the nuclear force, we will need an additional quantum number (or label) to indicate whether the nucleon is a proton or a neutron. Due to this formal analogy with ordinary spin, this label is called the isotropic spin or, in short, the isospin. If we take isospin quantum number as ½, its z-component will then be represented by a quantum number having the values ½ and -1/2. The difference between a proton and a neutron then becomes analogus to the difference between spin-up and spin-down particles.

The fundamental difference between ordinary spin and isospin is that, unlike the spin the isospin has nothing to do with rotations or spinning in the coordinate space hence cannot be coupled with the angular momenta of the nucleons. Nucleons can thus be distinguished by $\langle I_3\rangle = \pm 1/2$, where I_3 is the third or z- component of isospin vector. Now lets calculate the isospin numbers for the above elementary particles of the above reaction i.e, $p + p \rightarrow \pi^{+} + d$

$$n + p \rightarrow \pi^{0} + d$$

since multiplet number M is therefore assigned to such particles to indicate the number of their different charge states. For instance, for nucleons (protons and neutrons) M = 2, as M = 2I+1 then for nucleons by putting M = 2 we have, $2 = 2I+ 1 \Rightarrow I = ½$ so $I_3 = +1/2$ or – ½. So +1/2 is for proton and -1/2 is for neutron.

Similarly for pions, multiplet number M = 3, so $3 = 2I +1 \Rightarrow I = 1$ so $I_3 = +1, 0, -1$

+1 is for π^{+}, -1 is for π^{-} and 0 for π^{0}.

Now using this isospin quantum number we can calculate cross section.

$|pp\rangle = |I , I_3\rangle\, |I , I_3\rangle = |\frac{1}{2},\frac{1}{2}\rangle\, |\frac{1}{2},\frac{1}{2}\rangle = |1,1\rangle$ ……………….(1)

$|\pi^{+}d\rangle = |1,1\rangle\, |0,0\rangle = |1,1\rangle$…………….(2)

$|np\rangle = |\frac{1}{2},-\frac{1}{2}\rangle\, |\frac{1}{2},\frac{1}{2}\rangle$ …………..(3)

$|\pi^{0}d\rangle = |1,0\rangle\, |0,0\rangle = |1,0\rangle$…………..(4)

In order to calculate eqn (3) we have to use CG coefficients by taking isospin as angular momentum let $j_1 = ½$, $j_2 = ½$ then $J =|j_1-j_2|……|j_1+j_2|$

Lets consider another set of reactions

(1) $\pi^+ p \rightarrow \pi^+ p$
(2) $\pi^- p \rightarrow \pi^- p$
(3) $\pi^- p \rightarrow \pi^0 n$

we have to calculate the ratio of cross section using isospin

(1) solution: - first consider the isospin of all the particles present in three reactions i.e, π^+, π^-, π^0, n and p.

since multiplet number of pions is M= 3 so, according to formula 2I + 1 = 3 ⇒ I = 1 so I_3 = +1, 0, -1
+1 is for π^+, -1 is for π^- and 0 for π^0.
for nucleons (protons and neutrons) M = 2, as M = 2I+1 then for nucleons by putting M = 2 we have, 2 = 2I+ 1 ⇒ I = 1/2 so I_3 = +1/2 or – ½. So +1/2 is for proton and -1/2 is for neutron.

First consider $|\pi^+ p\rangle = |1, 1\rangle |\frac{1}{2},\frac{1}{2}\rangle = |\frac{3}{2},\frac{3}{2}\rangle$

$|\pi^- p\rangle = |1, -1\rangle |\frac{1}{2},\frac{1}{2}\rangle$………(A)

$|\pi^0 n\rangle = |1, 0\rangle |\frac{1}{2},-\frac{1}{2}\rangle$…………(B)

In order to calculate eqn (A) and (B) we use CG coefficients.

$j_1 = 1, j_2 = \frac{1}{2}, J = |j_1-j_2|\ldots\ldots|j_1+j_2| = \frac{1}{2},\frac{3}{2}$

For $J = \frac{1}{2}$ $M = -\frac{1}{2},\frac{1}{2}$ and for $J = \frac{3}{2}$, $M = -\frac{3}{2},\frac{3}{2}$

$|\frac{3}{2},\frac{3}{2}\rangle = |1, 1\rangle |\frac{1}{2},\frac{1}{2}\rangle$…………(1)

$|\frac{3}{2},-\frac{3}{2}\rangle = |1, -1\rangle |\frac{1}{2},-\frac{1}{2}\rangle$………(2)

$J_-|j\ m\rangle = \sqrt{(j+m)\ (j-\ m+1)}\ |\ j, m-1\rangle$…………(3)

Now applying J_- on eqn (1) : $J_- |\frac{3}{2},\frac{3}{2}\rangle = J_- |1, 1\rangle |\frac{1}{2},\frac{1}{2}\rangle$

$\Rightarrow\sqrt{3}\ |\frac{3}{2},\frac{1}{2}\rangle = \sqrt{2}\ |1, 0\rangle |\frac{1}{2},\frac{1}{2}\rangle + |1, 1\rangle |\frac{1}{2},-\frac{1}{2}\rangle$

[applying eqn (3)]

$\Rightarrow|\frac{3}{2},\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}\ |1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}\ |1, 1\rangle |\frac{1}{2},-\frac{1}{2}\rangle$…….(4)

$J_+ |j\ m\rangle = \sqrt{(j-m)(j+\ m+1)}\ |\ j, m-1\rangle$………(5)

Now applying j_+ on eqn (2) : $J_+ |\ j, m\rangle = J_+ |\frac{3}{2},-\frac{3}{2}\rangle = J_+ |1,-1\rangle |\frac{1}{2},-\frac{1}{2}\rangle$

$\Rightarrow |\frac{3}{2},-\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}\ |1,0\rangle\ |\frac{1}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}\ |1, -1\rangle |\frac{1}{2},\frac{1}{2}\rangle$…….(5)

[applying eqn (5)]

$|\frac{1}{2},\frac{1}{2}\rangle = \alpha\ |1, 1\rangle |\frac{1}{2},-\frac{1}{2}\rangle + \beta\ |1, 0\rangle |\frac{1}{2},\frac{1}{2}\rangle$

$J_+ |\frac{1}{2},\frac{1}{2}\rangle = \alpha\ J_+ |1, 1\rangle |\frac{1}{2},-\frac{1}{2}\rangle + \beta\ J_+ |1,0\rangle |\frac{1}{2},\frac{1}{2}\rangle$

$\Rightarrow 0 = \alpha\ |1, 1\rangle |\frac{1}{2},\frac{1}{2}\rangle + \beta\sqrt{2}\ |1,1\rangle |\frac{1}{2},\frac{1}{2}\rangle$ [applying eqn (5)]

Which shows, $\alpha + \beta\sqrt{2} = 0$

$\Rightarrow \beta = -\frac{\alpha}{\sqrt{2}}$

and $\alpha^2 + \beta^2 = 1$

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$\Rightarrow \alpha^2 + (-\frac{\alpha}{\sqrt{2}})^2 = 1 \Rightarrow \alpha = \sqrt{\frac{2}{3}}$

So now $\beta = -\sqrt{\frac{1}{3}}$

$\Rightarrow|\frac{1}{2},\frac{1}{2}> = \sqrt{\frac{2}{3}}\,|1,1>|\frac{1}{2},-\frac{1}{2}> - \sqrt{\frac{1}{3}}\,|1,0>|\frac{1}{2},\frac{1}{2}>$..........(6)

And $|\frac{1}{2},-\frac{1}{2}> = -\sqrt{\frac{2}{3}}\,|1,-1>|\frac{1}{2},\frac{1}{2}> + \sqrt{\frac{1}{3}}\,|1,0>|\frac{1}{2},-\frac{1}{2}>$...........(7)

Subtracting, $\sqrt{2}\times$ eqn (4) from eqn (2) we have, $|1,-1>|\frac{1}{2},\frac{1}{2}> = \sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}> - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}>$

Again adding eqn (7) with $\sqrt{2}\times$ eqn (5) we have $|1,0>|\frac{1}{2},-\frac{1}{2}> = \sqrt{\frac{2}{3}}\,|\frac{3}{2},-\frac{1}{2}> + \sqrt{\frac{1}{3}}\,|\frac{1}{2},-\frac{1}{2}>$

Using these two eqns in eqn(A) and eqn (B) we have

$|\pi^- p> = |1,-1>||\frac{1}{2},\frac{1}{2}> = \sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}> - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}>$

$|\pi^0 n> = |1,0>|\frac{1}{2},-\frac{1}{2}> = \sqrt{\frac{2}{3}}\,|\frac{3}{2},-\frac{1}{2}> + \sqrt{\frac{1}{3}}\,|\frac{1}{2},-\frac{1}{2}>$

$|\pi^+ p> = |1,1>|\frac{1}{2},\frac{1}{2}> = |\frac{3}{2},\frac{3}{2}>$

Because of charge independence in strong interaction, we can write

$<\frac{3}{2}, m_j|\hat{H}|\frac{3}{2}, m_i> = a_1,$

$<\frac{1}{2}, m_j|\hat{H}|\frac{1}{2}, m_i> = a_2,$

Independent of the value of m. furthermore the orthogonality of the wave functions requires

$<\frac{1}{2}|\hat{H}|\frac{3}{2}> = 0.$

Hence the transition cross sections are

$\sigma_1(\pi^+ p \rightarrow \pi^+ p)\ \alpha\ |<\frac{3}{2},\frac{3}{2}|\hat{H}|\frac{3}{2},\frac{3}{2}>|^2$

$\sigma_2(\pi^- p \rightarrow \pi^- p)\ \alpha\ |(\sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}> - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}|)\,\hat{H}\,(\sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}> - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}>)|^2$

$= |\frac{2}{3}a_2 + \frac{1}{3}a_1|^2$

$\sigma_3(\pi^- p \rightarrow \pi^0 n)\ \alpha\ |(\sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}> - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}|)\,\hat{H}\,(\sqrt{\frac{2}{3}}\,|\frac{3}{2},-\frac{1}{2}> + \sqrt{\frac{1}{3}}\,|\frac{1}{2},-\frac{1}{2}>)|^2$

$= |-\frac{\sqrt{2}}{3}a_2 + \frac{\sqrt{2}}{3}a_1|^2$

When resonance takes place $|a_1| >> |a_2|$, and the effect of a_2 can be neglected. Hence

$\sigma_1\ \alpha\ |a_1|^2,$

$\sigma_2\ \alpha\ \frac{1}{9}|a_1|^2$

$\sigma_3\ \alpha\ \frac{2}{9}|a_1|^2$

And $\sigma_1 : \sigma_2 : \sigma_3 = 9 : 1 : 2.$

When N^* resonance occurs, $|a_1| \ll |a_2|$, and we have

$\sigma_1 \approx 0,$

$\sigma_2\ \alpha\ \frac{4}{9}|a_2|^2$

$\sigma_3\ \alpha\ \frac{2}{9}|a_2|$

And $\sigma_1 : \sigma_2 : \sigma_3 = 0 : 2 : 1.$

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VII. RESULT AND DISCUSSION

We have calculated the cross sections for strong interactions and find their ratios.Clebsch Gordon co-efficients in isospin spin coupling are written in O(3) form. Future work is to find out the decay modes(branching ratio) of the interactions having applications in radiation physics.

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